Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0123456789

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **For examination from 2020**

SPECIMEN PAPER **1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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1 Find the set of values of x for which $3(2^{3x+1}) < 8$. Give your answer in a simplified exact form. [3]

2 **(a)** Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

(b) State the set of values of x for which the expansion is valid. [1]

3 **(a)** Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $3x - 1 > |2x - 3|$. [3]

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4 The parametric equations of a curve are

$$x = e^{2t-3}, \quad y = 4\ln t,$$

where $t > 0$. When $t = a$ the gradient of the curve is 2.

(a) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [4]

(b) Verify by calculation that this equation has a root between 1 and 2. [2]

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(c) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

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5 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x - \tan^{-1}x) = \frac{x^2}{1 + x^2}$. [2]

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(b) Show that $\int_0^{\sqrt{3}} x \tan^{-1}x \, \mathrm{d}x = \frac{2}{3}\pi - \frac{1}{2}\sqrt{3}$. [5]

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6 The complex numbers $1 + 3\mathrm{i}$ and $4 + 2\mathrm{i}$ are denoted by u and v respectively.

(a) Find $\frac{u}{v}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(b) State the argument of $\frac{u}{v}$. [1]

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In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $u - v$ respectively.

(c) State fully the geometrical relationship between OC and BA. [2]

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(d) Show that angle $AOB = \frac{1}{4}\pi$ radians. [2]

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7 **(a)** By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - \sqrt{2}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(b) Hence solve the equation

$$\cos(x + 45°) - \sqrt{2}\sin x = 2,$$

for $0° < x < 360°$. [4]

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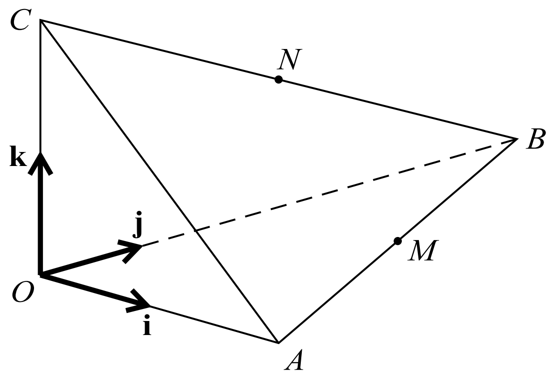

In the diagram, $OABC$ is a pyramid in which $OA = 2$ units, $OB = 4$ units and $OC = 2$ units. The edge OC is vertical, the base OAB is horizontal and angle $AOB = 90°$. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OC respectively. The midpoints of AB and BC are M and N respectively.

(a) Express the vectors $\overrightarrow{ON}$ and $\overrightarrow{CM}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

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(b) Calculate the angle between the directions of $\overrightarrow{ON}$ and $\overrightarrow{CM}$. [3]

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(c) Show that the length of the perpendicular from M to ON is $\frac{3}{5}\sqrt{5}$. [4]

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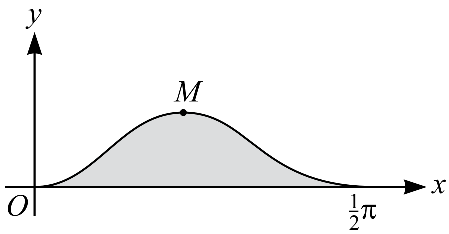

The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Find the x-coordinate of M. [6]

(b) Using the substitution $u = \sin x$, find the area of the shaded region bounded by the curve and the x-axis. [4]

10 In a chemical reaction, a compound X is formed from two compounds Y and Z.

The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$.

(a) Show that x and t satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.01(10 - x)(20 - x).$$ [1]

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(b) Solve this differential equation and obtain an expression for x in terms of t. [9]

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(c) State what happens to the value of x when t becomes large. [1]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.